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Comment

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Measuring Centrality and Power Recursively in the World City Network: A Reply to Neal

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Abstract

In a recent article, Zachary Neal (2011) distinguishes between centrality and power in world city networks and proposes two measures of recursive power and centrality. His effort to clarify oversimplistic interpretations of relational measures of power and position in world city networks is appreciated. However, Neal's effort to innovate methodologically is based on theoretical reasoning that is dubious when applied to world city networks. And his attempt to develop new measures is flawed since he conflates 'eigenvector centrality' with 'beta centrality' and then argues that 'eigenvector-based approaches' to recursive power and centrality are ill-suited to world city networks. The main problem is that his measures of 'recursive' centrality and power are not recursive at all and thus are of very limited utility. It is concluded that established eigenvector centrality measures used in past research (which Neal critiques) provide more useful gauges of power and centrality in world city networks than his new indexes.

Introduction

As scholars interested in world city network research, we were intrigued by Zachary Neal's article, "Differentiating centrality and power in the world city network". For decades, urban researchers have focused on systems of cities (McKenzie, 1927; Ross, 1987), frequently emphasising the 'dominance' of key nodes in these networks; more recently, key theorists have identified 'world cities' or 'global cities' as key 'basing points'

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and centres of 'command-and-control' for global capitalism (Friedmann, 1986; Sassen, 1991). This led to considerable empirical research using network analytical techniques to study the world city network (see Smith and Timberlake, 2001; Alderson and Beckfield, 2004; Taylor, 2004).

In most of this research, the notion that 'centrality' is related to 'power' is crucial; but Neal questions any assumption that the two are equivalent. Unfortunately, while we deeply appreciate Neal's effort to clarify oversimplistic interpretations of world city networks, we take issue with his attempt to differentiate centrality and power, his critique of eigenvector centrality and his alternative measures.

Conceptual Confusion

A clear contribution of Neal's paper is his recognition that 'centrality' and 'power' are not necessarily synonymous and that both depend not only on the ties a focal actor has with its neighbours, but also on the ties the actors in its neighbourhood have with their alters. In so doing, Neal differentiates between two kinds of centrality—one in which an actor is tied to other actors who are themselves central, and another in which an actor is tied to other actors who are themselves isolated. He refers to the former as recursive centrality and the latter as recursive power.

Yet, his discussion of 'power' and 'centrality' eventually falls flat. For example, he offers a graph of two hypothetical world city networks (Neal, 2011, Fig. 1) that illustrate one simple network (network B) with a central point and three connected nodes on spokes and a larger network (network A) in which a similarly positioned central point is connected to other nodes that are, in turn, connected to three more points. Neal claims that the central node in the smaller network is "powerful, but not central" and the centre of the larger network is "central, but not powerful". Yet, the "powerful" actor in network B is also the most central by any measure of network centrality as this is a perfect star graph. In short, while we agree that not all central actors are powerful and that both 'power' and 'centrality' should be considered recursively, Neal does not effectively demonstrate that powerful actors won't also be central (also see Allen, 2010; Bonacich, 1987).

Moreover, the notion of power advocated by Neal draws heavily from exchange theory (for example, Neal, 2011, p. 2736). While we take no issue with exchange theoretic conceptualisations of power in general, we find them odd in the empirical case of many world city networks. Let us imagine two cities in the case of between-city air passenger flows—city A and city B. City A is central in the recursive sense: it is not only connected to many cities, but also to many cities that are themselves central. City B is connected to many cities that are only connected to city B. If cities A and B were engaging in barter trade, city B would clearly have more power to determine the"exchange ratio'' (i.e. the ratio of goods sent to goods received) with its neighbourhood. However, in a network of air passenger

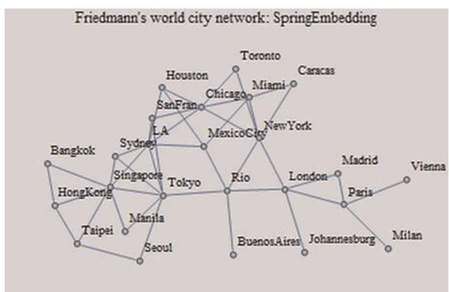

Figure 1. Spring embedding of Friedmann city graph.

flows, it is not clear that cities are exchanging anything where bargaining power would matter; that is, it is not entirely clear what is being bargained. One can ask similar questions with respect to intercity bandwidth connections, headquarter subsidiary relations, etc. In short, we are sceptical that bargaining power in exchange networks is the only way in which power can be conceptualised for world city networks; indeed, it may not make much sense in many empirical contexts for these relations—a point that may extend to research on world-city networks beyond Neal's.

Early in his paper Neal focuses on degree centrality, claiming that it is "the most widely employed measure" used to gauge world city networks and noting that it simply involves "counting a city's total number of linkages" (Neal, 2011, p. 2737). While there is published analysis that uses this measure, many key studies use various types of valued and/or 'recursive' measures including eigenvector centrality (Smith and Timberlake, 2001), 'betweenness' and 'closeness' (Alderson and Beckfield, 2004). Although Neal mentions these additional techniques in passing he summarily dismisses this family of recursive measures as 'inappropriate' for world city networks for various reasons. The remainder of his article contrasts his 'recursive' centrality measures to simple degree centrality. He thus proceeds from an assumption that established recursive centrality indices, such as eigenvector and betweenness, are inappropriate to world city networks, which we show to be erroneous.

Measuring Power and Centrality Recursively: A Re-analysis of Friedmann Based on Eigenvector and 'Recursive' Centrality

The major thrust of our concern with the paper deals with the measurement of centrality and power, recursively. First, Neal repeatedly uses the phrase 'eigenvector-based measures of centrality', but it is not entirely clear what he means. He cites two seminal papers by Bonacich, one introducing 'eigenvector centrality' and the other introducing 'beta centrality', which are both conceptually and computationally unique. In any case, he seems to believe that both are inappropriate to world city networks for two reasons. On the one hand, such measures are inappropriate to networks with a high degree of 'clustering', which appears to be the case when the second eigenvalue is "large" relative to the first (p. 2740). On the other hand, "eigenvector-based measures" of centrality are allegedly inappropriate to "large" networks, or to networks with many strong ties, which leads to a situation in which the first eigenvalue is "too large" (Neal, 2011, p. 2740). We show that not only are Neal's measures of 'recursive' power and centrality hardly any more recursive than degree centrality, but also that his criticisms of eigenvector centrality (as opposed to beta centrality) are unfounded.

First, we must point out that his two 'recursive' formulas are in no sense 'recursive'. An example of a recursive definition is that of the factorial function, where $n!$ is defined in terms of smaller values of the function: $n! = n(n–1)!$. To avoid an infinite regress, we must define $0! = 1$. Another example is the definition of a (general) *tree* as a root to which is attached a (possibly empty) sequence of trees. By way of contrast, his recursive centrality is defined in one step

$$\mathbf{R}C_i = \sum_i R_{ij} DC_j \tag{1}$$

If we use c_0 for degree centrality and c_1 for 'recursive' centrality we see that equation (1) can be expressed as a matrix times a vector

$$c_1 = \mathbf{R}c_0 \tag{1a}$$

In fact, degree centrality itself can be expressed as the product **R1**, where **1** is the vector of all 1s. If we continue this process, the numbers in the vectors get either too large or too small, so to keep them in the middle range divide by the Euclidean norm of the vector, so we have

$$c_{k+1} = \frac{\mathbf{R}c_{k-1}}{\|c_{k-1}\|}. \tag{1b}$$

Since this process is known to converge, we will find a large enough integer n such that c_n is close to c_{n-1}. When this occurs, we know that $\|\mathbf{R}c_{k-1}\|$ is approximately equal to the largest eigenvalue λ_1 and that c_n is approximately equal to its eigenvector. It is customary to normalise the eigenvector to be of unit length (and not make the largest value equal to 1 as did Neal). While this is a terrible way to compute an eigenvalue (compared with, say, the 'shifted inverse power method' (Strang, 1988)), it does put three of the centrality measures in perspective: degree centrality is the first-order approximation to eigenvector centrality, while 'recursive centrality' is the second-order approximation. Convergence is slow, because of the relatively large second eigenvalue, but after 30 iterations, we found that the root mean square error for the normalised eigenvector was 0.0000055, while $\|Ac_k\|$ was only 7.53×10^{-10} smaller than the eigenvalue λ_1.

We display a re-analysis of the Friedmann data and compare degree, 'recursive centrality' (reCent), "recursive power" (rePower) and eigenvector centrality (eigen) in normalised form in Table 1. In doing so, we first had to unpack some discrepancies between Neal's Figure 2 and Table 1 (Neal, 2011, p. 2741). Using the original graph of "Friedmann's world city network": Tokyo is of degree 7, not 6, and "Rio/São Paulo" is of degree 5, while Neal's figure has a node labelled "São Paulo" of degree 3 with the corresponding label in Table 1 being "Rio de Janiero" of degree 5. (Also, he omits links to Tokyo, Mexico City and New York, while putting in a link to Caracas that does not appear in Friedmann's Figure 2.)

In our initial critique of Neal submitted to the journal, we assumed (as no doubt most readers would) that the matrix he analysed was identical to the erroneous reproduction of the Friedmann graph. However, we learned later that Neal's Table 1 reports centrality scores for Friedmann's original graph. So, we recalculated based on Friedmann's original figure, as drawn in our Figure 1 with standard "spring embedding". To resolve the Rio/São confusion, we use the label "Rio" throughout.

Note that Singapore starts out with a degree centrality of 0.354, then falls to 0.308 in the second-order approximation (i.e. 'recursive centrality'), before increasing again to its limiting eigenvector value of 0.341. This is because two of its neighbours, Bangkok and Manila, have a relatively low degree of 2. Other cities show a different pattern because the eigenvector takes account of *all* possible long-distance walks on the graph, rather than simply the degree of each member of a city's neighbourhood. This analysis is further confirmed by considering in Table 2 the correlations between the measures: degree is highly correlated with 'recursive centrality', which in turn is even more highly correlated with eigenvector centrality. Finally, degree and eigenvector centralities are less highly correlated than are 'recursive centrality' and degree. It appears that, rather than eigenvectors, 'recursive centrality' is "equal or nearly equal to … ordinary degree centrality" (Neal, 2011, p. 2743).

Also notice in Table 2 that 'recursive power' has a low correlation with the

Table 1. Normalised centrality measures of Friedmann cities

City	*Degree*	*reCent*	*eigen*	*rePower*
Los Angeles	0.354459	0.37607	0.425756	0.269955
Singapore	0.354459	0.307694	0.340879	0.362457
Tokyo	0.354459	0.330486	0.380029	0.341314
Chicago	0.303822	0.284901	0.305221	0.273731
New York	0.303822	0.273505	0.223908	0.288078
London	0.253185	0.205129	0.116768	0.335651
Rio	0.253185	0.262109	0.216647	0.279017
Mexico City	0.202548	0.216525	0.225663	0.146871
Miami	0.202548	0.205129	0.183967	0.17179
Paris	0.202548	0.102565	0.03617	0.428153
Hong Kong	0.151911	0.136753	0.123742	0.1548
Houston	0.151911	0.193733	0.208587	0.088727
San Francisco	0.151911	0.227921	0.242245	0.071736
Sydney	0.151911	0.239317	0.25002	0.067961
Taipei	0.151911	0.136753	0.125332	0.1548
Bangkok	0.101274	0.113961	0.101307	0.075512
Caracas	0.101274	0.113961	0.088933	0.066073
Madrid	0.101274	0.102565	0.033347	0.071359
Manila	0.101274	0.159545	0.157188	0.045307
Seoul	0.101274	0.113961	0.110189	0.075512
Toronto	0.101274	0.136753	0.115372	0.052858
Buenos Aires	0.050637	0.05698	0.047238	0.031715
Johannesburg	0.050637	0.05698	0.02546	0.031715
Milan	0.050637	0.045584	0.007887	0.039644
Vienna	0.050637	0.045584	0.007887	0.039644

Table 2. Correlations between centrality measures

	Degree	*reCent*	*eigen*	*rePower*
degree	1	0.918785	0.846041	0.869256
reCent	0.918785	1	0.968133	0.632839
eigen	0.846041	0.968133	1	0.50975
rePower	0.869256	0.632839	0.50975	1

previous three. The last column of Table 1 is a normalised listing of recursive power values defined by the formula

$$\mathbf{R}P_i = \sum_j \frac{R_{ij}}{DC_j} \qquad (2)$$

This expresses a kind of 'relative power' for a node which increases as its degree increases and as the degrees of its neighbouring nodes decrease. For a given degree, the maximum 'recursive power' is when each node in its neighbourhood is 1: the star graph. Like the 'recursive centrality' index, its (un-normalised) value is a function only of its local neighbourhood, and not of the graph as a whole. Hence, they both (along with degree centrality) will have limited interest in network studies on world cities. The indices that will have more interest, because they reflect the network structure as a whole, include betweenness and eigenvector centralities. As an aside, betweenness centrality has been generalised to valued data, so that, contrary to Neal's claim, it is perfectly suitable to world city networks with valued ties (Freeman *et al*., 1991).

It is interesting to note that, if 'recursive power' is iterated, as we did with 'recursive centrality', then there is extremely slow convergence to a vector with a single 1 (for Paris) with the rest being 0. London is the last to approach 0: after 1000 iterations, its value is 0.002, while the third-largest city has a value of 2×10^{-150}. While the French might like the idea of Paris being omnipotent, this approach does not seem promising to us.

Erroneous on Eigenvectors

One target of Neal's critique is eigenvector centrality measures that were employed in recent work on world city networks (for example, Choi *et al*., 2006; Mahutga *et al*., 2010; Smith and Timberlake, 2001). To show that these sorts of measures do not work with real data, he provides information on international city-to-city bandwidth capacity on the Internet. He says (p. 2743) of the internet data "eigenvector-based measures … are not appropriate because the network's largest eigenvalue (λ_1 = 178231.55) is too large".

However, this concern is misguided. The absolute size of the first eigenvalue has absolutely no bearing on the utility of eigenvector centrality. The proof is simple—if λ is an eigenvalue for the vector **v** and matrix A, then $\mathbf{A}\mathbf{v} = \mathbf{v}\lambda$. Changing units is equivalent to multiplying both sides by a scalar *c*, giving $c\mathbf{A} = v(c\lambda)$, showing that $c\lambda$ is an eigenvalue for $c\mathbf{A}$, leaving the eigenvector unchanged.

The second issue Neal raises concerns the size of the second eigenvalue relative to the first, which is presumably related to his other concern about networks with too much 'clustering'. He states (p. 2740) that "eigenvector-based measures … are not appropriate because the network's second largest eigenvalue ($\lambda_2 = 3.45$) is relatively large compared with its first eigenvalue ($\lambda_1 = 4.59$)". There is still no need to discard all these results; instead, it suggests an examination of the second eigenvector as well.

In the vaster literature on spectral analysis of graphs, many of the eigenvalues are interesting, including the second-largest. The difference between the first and the second-largest eigenvalues is called 'the spectral gap' and is related to the speed of convergence of Markov processes. It is also important in the computation of eigenvalues by an obsolete method known as the 'power method', which is related to Neal's 'recursive centrality'. If the first eigenvalue is at all larger than the second eigenvalue, then the first eigenvector will give the best-fitting global 'recursive' measure of centrality except when the network consists of disconnected components, in which case each actor on the smaller of the two components will have zeros on the first eigenvector. However, the second eigenvector will return zeros for the larger component and non-zero values for the second. Thus, we see little value added by his 'recursive' measures even in this case, since they are presumably motivated by a desire for a global measure of centrality. Indeed, if there is more than one component (i.e. two networks instead of one), one should consider measuring centrality separately for each. Doing so in the context of eigenvector centrality is well understood and relatively straightforward—one simply examines separate eigenvectors for each component, which will be conveniently weighted by their respective eigenvalues in a full eigen decomposition.

Conclusion

Some of Neal's general points and conclusions are right. World city networks are multiplex and any attempt simply to equate 'centrality' with the types of 'power' that are critical to conceptual understandings of world/global cities should be discouraged. We also agree that "the conception of world cities' status in networks as a unidimensional, hierarchical phenomenon" (p. 2745) is misguided and oversimplified. Thus, recursive conceptualisations of power and centrality are often more meaningful than non-recursive ones; if we take networks seriously, then we should incorporate information from the whole network in our attempts to measure centrality and power. Doing so will provide a more realistic understanding of global city networks. Yet, we also think that the meaning of power varies by the relation being analysed—a point made rather persuasively in Bonacich's (1987) intervention. Moreover, Neal's measures of 'recursive centrality' and 'recursive power' are not very recursive—the only additional information they provide is limited to the degree of an actor's local ties, rather than the ties to those ties, and their ties and their ties, etc. Finally, we are much more confident in the utility of eigenvector centrality as implemented in various studies of the world city network, which provide a fruitful way to measure the power and centrality of cities in the world city system.

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